Report on 29th Novembertagung on the History of Mathematics

Co-sponsored by the ICHM

28th-30th November 2018

The 29th Novembertagung on the History of Mathematics took place at the Institute of Mathematics of the University of Sevilla (Spain) on November 28th to 30th. The theme we proposed was "History of Mathematical Concepts and Conceptual History of Mathematics". As is usual in the Novembertagung, the theme served as a guide and did not preclude the participation of thosenot committed to it directly.

This edition of the Novembertagung counted 45 participants from 13 different countries: Spain, United Kingdom, Mexico, Germany, France, Israel, Portugal, Switzerland, USA, Belgium, Austria, Italy and the Czech Republic. Although the majority of participants were coming from Europe, some of them came from other continents as well, which gives an idea of the international level of the event and the wider diffusion that the Novembertagung is gaining year after year. This opens the possibility of future venues outside Europe and suggests that history of mathematics is very alive among young researchers.

There were 35 talks divided into two parallel sessions and two plenary sessions given by senior scholars (see the program and the abstracts below). As is traditional in the Novembertagung, there was a final meeting to discuss next year's venue and some other issues participants wished to raise relating to the future of the event.

As an event for young researchers, thanks to the help given by several sponsors, we were able to cover food and accommodation for every participant during the meeting and we were able to give a high number of travel grants for those who requested them. At the end of this document, all the sponsors, who helped us, together with the ICHM are listed. I want to express my gratitude to all of them, as well as to the other members of the organizing committee, who are also listed below.

María de Paz, Universidad de Sevilla (Spain)

Chair of the 29th Novembertagung Committee

Organizing Committee of the 29th Novembertagung.

Eduardo Dorrego López, Universidad de Sevilla (Spain)

Manuel J. García-Pérez, Universidad de Sevilla (Spain)

Elisa Turiello Hernández Universidad de Sevilla (Spain)

Elías Fuentes Guillén, UNAM (Mexico)

Brigitte Stenhouse, Open University (UK).

Official Picture of the Participants, Novembertagung 2018, Sevilla (Spain).

Program

29th Novembertagung on the History of Mathematics

“History of Mathematical Concepts and Conceptual History of Mathematics”

28-30 Noviembre 2018 Instituto de Matemáticas de la Universidad de Sevilla (IMUS)

NOVEMBER 28th		
12h Registration		
13h lunch		
14h 30 Conference Opening – Room: Salón de Actos		
15h parallel session 1	SESSION A – Room: Salón de Actos. Chair: Brigitte Stenhouse	SESSION B – Room: Seminario I. Chair: Rosie Lev-Halutz
	-P. Debroise – Configurations - M. Muffato – Quest for practical arithmetics III: Are they really what we think they are ? - E. Fuentes Guillén - Conflicting Mathematical Objects c. 1800	- A. Linares – The sheaf of Jean Leray and the sheaf of Henri Cartan -E. Rinaldi - Henri Cartan’s contribution to the theory of potential during the Second World War - S. Duran -1900-1923. E. J. Wilczynski and the field of Projective Differential Geometry: how a privileged actor at national level circulate his research at international level?
16h45 Break		
17h15 parallel session 2	SESSION A – Room: Salón de Actos. Chair: María de Paz	SESSION B – Room: Seminario I. Chair: David Waszeck
	- E. Dorrego López – From that which transcends algebra to the non-algebraic. The case of e and π as motor for the establishment of the modern notion of transcendence - A. Parker - Newton’s mathematical concept of force - D. Molinini - Mathematical Concepts as Conceptual Mediators	- D. Koenig - Ernst Cassirer and the imaginary in mathematics – An alogical moment in the evolution of mathematics? - H. Heller - Concepts of group theory - G. Ricotier - Transition of the style of mathematical publications during the 20th century
19h End day 1		
20h dinner at the residence		

NOVEMBER 29TH		
10h plenary lecture – Room: Salón de Actos. Chair: Manuel J. García Pérez	Karine Chemla - *Fragments of a history of the concept of ideal. Poncelet and Chasles' reflections on generality in geometry and their impact on Kummer's work with ideal divisors*	
11h15 break		
11h45 parallel session 3	SESSION A – Room: Salón de Actos. Chair: Elías Fuentes Guillén	SESSION B – Room: Seminario I. Chair: Colin Rittberg
	- P. Seban – From Zeno *ad infinitum*, iterative reasoning in early Greek mathematics -B. Wilck- Euclid's Definitions	- J. Pérez Escobar - Mathematical modelling in biology and non- materialistic idealizations: the resilience of the gap between biology and physics - R. Kelter - Markov-Chain-Monte- Carlo for Hypothesis Testing – An alternative to p-values with regards to the replication crisis of medical studies
13h lunch		
14h30 parallel session 4	SESSION A – Room: Seminario II. Chair: Brigitte Stenhouse	SESSION B – Room: Seminario I. Chair: Nicolas Michel
	- M. J. García-Pérez - The Prehistory of geometry: Early China as a case study - T. Hirth – From Pebbles to Nimbers and Stars - M. Chopra - Before or beyond concepts	- J. Zeman - Hilbert's Grundlagen der Geometrie in context of non- Euclidean geometries -S. Shokrani – Leonard Nelson's Philosophy of mathematics and its Application - T. Schütz - Exploring Gravitational Lensing
16h15 break		
16h45 parallel session 5	SESSION A – Room: Seminario II. Chair: Eduardo Dorrego	SESSION B – Room: Seminario I. Chair: Manuel J. García-Pérez
	- S. Pietroni – Equation of time in Alfonsine Astronomy -G. Loizelet - Sun-Earth distance: Al-Biruni facing a correct calculation method that cannot be trusted	-M. Friedman - Braids, diagrams, models and the concepts of 'Anschaulichkeit' and 'Intuition' - D. Waszek - On the use of the concepts of 'representation' in the history of mathematics
18h End day 2 – Official Group Picture		
20h dinner at the residence		

NOVEMBER 30TH		
9h30 Parallel session 6	SESSION A – Room: Seminario II. Chair: Eduardo Dorrego	SESSION B – Room: Seminario I. Chair: Paul Hasselkuss
	- B. Stenhouse – Embracing Nature in Formulae: The Hidden Mathematics of Mary Somerville - A. Kiel Steensen – Dedekind's articulation of the Modulgesetz. A case study of a semiotic process in mathematical practice	-D. Kant - Forcing as a Part of Set- Theoretic Practice -M. Imocrante & M. Osta - Conceptual change and (Philosophy) of mathematics -C. J. Rittberg - Intellecual Humility in Mathematics
11h15 Break		
11h45 Parallel session 7	SESSION A – Room: Seminario II. Chair: Daniele Molinini	SESSION B – Room: Seminario I. Chair: Elías Fuentes Guillén
	-J. L. Gastaldi - The emergence of a mathematical concept of 'symbol' in the work of Robert Woodhouse - N. Michel - Numbers as concepts & numbers as symbols	-S. Decaens – Lattice theory, abstract algebra and abstraction in the 1930s -M. Chalmers - Georges Bouligand's concept of direct methods in mathematics
13h Lunch		
14h30 Plenary lecture – Room: Seminario II. Chair: María de Paz.	José Ferreirós (US) - Investigating long-term developments in the history of mathematics: The case of the function concept.	
15h45 break		
16h discussion – Room: Seminario II	The future of Novembertagung	
18h closing – Room: Seminario II.		
20h30 Final dinner		

29th Novembertagung on the History of Mathematics

History of Mathematical Concepts and Conceptual History of Mathematics

28-30 November 2018, Seville (Spain)

Book of Abstracts

WEDNESDAY 28 NOVEMBER

Parallel Session 1, 15:00h – SESSION A

P. Debroise - *Configurations*

Many fundamental concepts of analysis and classical mathematical physics are rooted in scholastic efforts to produce a mathematical science of alteration, in particular to measure the variation of "intensity" of quality and movement. One of the main results of this effort is Nicole Oresme's *Tractatus de Configurationibus qualitatum et motuum*, composed around 1350 in Paris, which I study currently. I propose to focus on the meaning of this mathematical elaboration, more precisely on the concept of "configuration".

Three main ideas can be generalized from this case study.

First, a new mathematical conception is not necessarily added to old materials, but can consist in the old materials reelaborated. Oresme's theory seems to be an application of geometry to alterations, but actually requires traditional finite *figures* of geometry to be read as potentially infinite *configurations* defined in terms of uniformity and difformity.

Second, an abstract concept can be hardly understandable without specification of new kinds of questions asked in old practices, even seemingly far from our own concerns. In Oresme's *Tractatus*, the configuration concept is motivated, among other things, by the necessity to solve esthetic problems about the new French polyphonic music.

Third, as neutral as it seems, the mathematical concept can serve more general and philosophical goals. Mathematical configurations are mainly justified by the impossibility to go beyond statistical knowledge about nature, giving thus an explanation of magical effects.

This study should make it clear, as I would argue, that the meaning of a new mathematical conception should not be considered without non-mathematical and even non-scientific materials which explain it.

M. Muffato - *Quest for practical arithmetics III: Are they really what we think they are?*

From the Middle-Ages and for centuries on, practical arithmetic has been a prolific field of production of mathematical treatises. However, the field "practical arithmetic" should not be considered properly defined. I will try to address this question and also study the structure of those texts.

The point of this presentation will be to explore a corpus of practical arithmetics written in French during the XVIIth century in order to answer the above two questions.

In my talk, I will investigate the content of those texts in order to formulate a definition of the field "practical arithmetic", and their texts. I will then use hints given by the author themselves especially in the treatises' forewords, taking their own interpretation of the existing field and tradition.

I will also discuss the relevance of labelling all those treatises as "practical arithmetic". Then I would like to give a new name to this type of treaty, and propose several sub-categories that would give more details on their content, and also match closer their historical tradition.

I will try to bring elements of answer to these questions both through a general analysis of the corpus and case-studies of some treatises.

E. Fuentes Guillén – *Conflicting Mathematical Objects c. 1800*

This talk will focus on two intertwined movements. On the one hand, at the turn of the 19th century there were still several mathematical concepts that awaited further clarification, such as the opposites, the irrationals and the imaginary. The status of these objects ranged from ‘numbers’ and ‘quantities’ to mere ‘expressions’, and while there was some consensus on what each of them was and why they were not something else, there were also discordant voices. That way, for example, while Johann Friedrich Schultz (professor at Königsberg) agreed with other contemporary Germanic authors on the fact that irrationals were numbers and warned against considering the negative quantities as unreal or defective (Schultz, 1790; Langsdorf, 1802; Zimmermann, 1805), Bernhard Friedrich Thibaut (professor at Göttingen) argued that expressions such as 7 should not be called numbers and used the name “conflicting numbers” to refer to the opposites (Thibaut, 1809).

On the other hand, the works of all those Germanic authors and some others show how new pedagogical concerns and methodological and foundational reflections arose. Schultz, for example, pointed out the need to pay more attention to the mathematical terms and concepts used, as well as their concatenation, and for this he was praised by Bolzano (Bolzano, 1810). But also Thibaut criticized the foundation of arithmetical theories on geometrical considerations and the use of “extraneous principles” that contravened the purity of analysis (Thibaut, 1805 & 1809), Christian Gottlieb Zimmermann (teacher in Berlin who, according to the ADB entry, was close to Schultz and Kant), intended to modify (with Pestalozzi as a reference) the usual way of explaining the whole and the opposite quantities (Zimmermann, 1805), and Johann Andreas Christian Michelsen (professor at the Berlinischen und Cöllnischen Gymnasium) quoted Kant’s idea of the construction of concepts to account for the innovations that he carried out for the sake of firmly established and well explained concepts, as those of quantity and variation (Michelsen, 1789).

Parallel Session 1, 15:00h – SESSION B

A. Linares - *The sheaf of Jean Leray and the sheaf of Henri Cartan*

In 1940 Jean Leray was sent to an officer’s camp in Austria. There, he offered a course in Algebraic Topology that was announced in the Comptes Rendus de l’Académie des Sciences, in 1942. It was not completely published until the war was over and it contained a new definition of cohomology. That was the frame where the concept sheaf was born.

In 1944, Henri Cartan used the concept that was built by Leray to analyze the ideals of holomorphic functions of n variables. Four years later, he organized a seminar. In the seance of 1948/1949 the notion of coherence was added and the definition was rewritten to a simpler form. In doing that, the concept became wider and started to resemble the one that we use today. Both definitions share a common notion, the passing of local data to a global property, but they were made in response to different questions.
In this work we analyze the differences between these two definitions, their origins and their purpose to understand a part of the evolution of one of the key concepts of modern mathematics.

E. Rinaldi - *Henri Cartan's contribution to the theory of potential during the Second World War*

Henri Cartan, a famous mathematician of the twentieth century, one of the founders of the Bourbaki group, has given new and important contributions to the theory of potential during the years of the Second World War. The talk wants to analyze in a historical key a research on this interesting aspect of the history of mathematics.

S. Duran - *1900-1923. E. J. Wilczynski and the field of Projective Differential Geometry: how a privileged actor at national level circulate his research at international level?*

At the beginning of the 20th century, E. J. Wilczynski began to work on projective differential geometry, and became part of a growing international field of research. After a university training in Germany (ending in 1900), Wilczynski rose in rank in different institutions of the USA: universities, mathematical societies and foundations. During this time, he also became one of the most published geometers of USA, until the end of his career for illness in 1923. Furthermore, from the 1930s historiography regularly have associated him with an "American school of projective differential geometry", a school of which he would have been the leader. I will start from these observations to analyze the specific place of US works in this field of research at the international level. I will focus on how to characterize the circulations of knowledge in projective differential geometry, from the USA to foreign countries, and the role played by Wilczynski in these processes. The purpose of my talk will be to understand how the privileged situation of E. J. Wilczynski allowed him to be presented as a leader of an internationally recognized research school of Projective Differential Geometry.

Parallel Session 2, 17:15h – SESSION A

E. Dorrego López - *From that which transcends algebra to the non-algebraic. The case of e and π as motor for the establishment of the modern notion of transcendence*

Over the course of the second half of the seventeenth century, the so called degeometrization process originated by the introduction of algebraic methods in geometry, led more and more the attention from the geometric part of the quadrature problems to the analytic one. This change of paradigm placed the focus into the search of finite combinations of suitable algebraic operations - those constructible by rule and compass - expressing the result. Now, the attention was paid to studying these formal expressions and therefore the values expressed by them on their own, some of which involved infinite processes as the case of the quadrature of the circle. It is in this context that Leibniz uses the term transcendence in the field of Mathematics, precisely to make reference to objects that in some way transcend algebra, and this is the ambiguous meaning inherited by authors like Euler (who uses it above all for functions). To find the first modern use of the term transcendental as the non-algebraic - from a modern point of view and therefore applied to quantites like e or π - we have to wait for Lambert and his Mémoire of 1768.

Throughout this work and after proving certain irrationality results, he emphasises to what extent his conclusions offer a glimpse of the transcendence of these quantities, hinting, as he did more specically at the end of the article, the necessity of distinguishing between irrational quantities: the algebraic quantitites - that is to say, all roots of algebraic equations - and the others. Therefore, in Lambert's Mémoire these Leibnizian transcendental quantities become the modern transcendental ones.

References:

E. Knobloch, Beyond Cartesian limits: Leibniz's passage from algebraic to transcendental mathematics, Historia Mathematica, Vol. 33, 2006, pp. 113131.

J. H. Lambert, Mémoires sur quelques propriétés remarquables des quantités transcendantes, circulaires et logarithmiques, Mémoires de l'Académie royale des sciences de Berlin, 1761/1768, pp. 265322.

Bruce J. Petrie, Johann Heinrich Lambert's Use and Understanding of Mathematical Transcendence, Conference 2011.

Bruce J. Petrie, Leonhard Euler's Use and Understanding of Mathematical Transcendence Historia Mathematica Vol. 39, 3, 2012, pp. 280291.

A. Parker - *Newton's mathematical concept of force*

Against Descartes' admission of algebraic curves into mathematical practice, Newton's Principia (16871, 17132, 17263) urges a return to the Euclidean paradigm where geometry is a part of 'general mechanics' and geometric objects are the results of quasi-causal motions (e.g. the rectilinear motion of a point generates a line segment). This position borrows from Isaac Barrow's position, in his Mathematical Lectures, that real definitions of geometric curves may postulate the mechanical causes (tracing mechanisms) of their generation. Though the Principia is deliberately divided into mathematical (Books I and II) and physical (Book III) portions, Newton clearly views this quasi-causal conception of mathematics as central to his method in deriving universal gravity in Book III. But how are the mechanical quasi-causes generating mathematical curves related to the physical causes generating real motions? Inspired by the scholarship of Niccolò Guicciardini and George Smith, I offer a reading connecting the two halves of the Principia – mathematical and physical – by illustrating how the success of Newton's method of successive approximations in Book III may depend upon some quasi-causal features of his conception of mathematics. By considering a historical debate in meta-mathematics about the classification of geometric curves, we can illuminate an important proof in the history of mathematical physics. I argue that Newton's more permissive study of curves produced by "any motions whatsoever" is a central to his generalization of the Galilean law of free-fall and his revolutionary conception of force as a mathematical magnitude with additive structure.

D. Molinini - *Mathematical concepts as conceptual mediators*

The problem of accounting for the success of mathematics in empirical applications has been addressed from many different standpoints. In this talk I propose an alternative way to attack the problem that largely hinges on historical considerations. I maintain that, as shown by some cases drawn from the history of science and the history of mathematics, what is central to a class of specific applications of mathematics in the empirical sciences is the function that mathematics has to embed empirical concepts and mediate between empirical settings that share the same (or almost the same) conceptual framework. It is in this mediating function that, at least for the specific cases I shall analyse, lie the reasons for the successful applicability of mathematics. Nevertheless, I shall also point to three main difficulties that such an analysis is confronted with. First, mathematics is subjected to conceptual shifts and embedding internally (i.e., within mathematics itself) and these conceptual revisions should be accounted for when we examine its success in application; second, the border between mathematics and what is mathematized can be very evanescent (substances dissolve "in the acid of mathematics", as Yves Gingras has remarked) and therefore it may be difficult to disentangle the mathematical content from the empirical content to which mathematics is applied. Third, different kinds of applications may (and, actually, seem to) require other strategies of analysis that share little with the treatment I propose in this talk.

Parallel Session 2, 17:15h – SESSION B

D. Koenig - *Ernst Cassirer and the imaginary in mathematics – An alogical moment in the evolution of mathematics?*

The notion of complex numbers is central for the development of mathematics in the 20th and 21st century. Nonetheless, it is relatively new and its first conceptualizations came up in the late 18th and 19th century after being considered as irrelevant or imaginary. One of the main concerns of the philosopher Ernst Cassirer's (1874-1945) in the field of philosophy and history of mathematics is to understand how it is possible that in the rigor and alleged ahistorical science of mathematics such a central notion can come up so late and revolutionize complete parts of mathematics.

The aim of my talk is to show that for Cassirer revolutionary developments, such as the invention of complex numbers, are grounded in mathematics itself and exemplifies so to speak alogical moments in the history of mathematics which indicates an antilogicism of Cassirer. Furthermore, I want to show that Cassirer's differentiation between mathematics and logic is central for his view on mathematics in the context of his general philosophy of culture.

H. Heller - *Concepts of group theory*

Algebraic structures like groups are axiomatically defined in terms of how their elements operate. Historically the group axioms mark a step of a (first order) abstraction of mathematical objects. Also philosophically, the abstract principles of group theory seem to be in line with a structuralist interpretation of mathematics. In mathematical practice, however, structures are often studied in terms of their respective sub- and superstructure properties, or in terms of their morphisms, where little attention is paid to the single elements of the structure. Central notions in group theory are (normal/characteristic/central/commutator.) subgroups and (central/stem/perfect) extensions and some of the main theorems of the theory are connected to classification and (de-)composition results.

There are a number of – implicit and explicit – attempts to comprise the "second order abstraction" of group theory that avoids the notion of group elements altogether. These systems, however, could never be formally axiomatised. In my presentation I want to follow the changing concept of group theory, focussing on three different attempts to implement an element-free notation of group theory, namely representation theory (from Klein [1884] and Schur [1911] to Noether [1929]), lattice theory (from Dedekind [1900] and Rottlaneder [1928] to Ore [1935,6]) and homological algebra (again Schur [1911] and much later Cartan, Eilenberg [1956] and Grothendieck [1957]).

At a philosophical level, the discrepancy between the "first order abstraction" of the classical axioms and the "second order abstraction" of mathematical practice provides an interesting case study for the fact that axiomatisation and abstraction do not always point into the same direction.

G. Ricotier - *Transition of the style of mathematical publications during the 20th century*

The talk will reflect on the evolution of the style of exposition in mathematical publications during the middle half of the 20th century, in particular under the influence of (members of) the Bourbaki group. The introduction and the unfolding of mathematical concepts changes during this period, and a focus of my study concerns the way in which this process transforms the mathematical production as a whole.

THURSDAY 29 NOVEMBER

Plenary Lecture, 10:00h

Karine Chemla - *Fragments of a history of the concept of ideal. Poncelet and Chasles' reflections on generality in geometry and their impact on Kummer's work with ideal divisors*

Kummer's first public presentation of the concept of "ideale complexe Zahlen" (published in the 1846 issue of the *Bericht über die zur Bekanntmachung geeigneten Verhandlungen der Königl. Preuss. Akademie der Wissenschaften zu Berlin*) draws a parallel between ideal elements in geometry and the "ideal prime factors" that Kummer introduces in his study of cyclotomy. In this first publication of his theory, Kummer also drew other comparisons with the introduction of complex numbers into "algebra and analysis", and Gauss's work in number theory, whereas in later expanded presentations he established a parallel with chemistry. The historiography of Kummer's work in number theory has mainly dwelled on these other comparisons, to account for Kummer's conceptual innovation. However, the parallel with projective geometry has remained in the shadow. In this talk, I argue that Kummer's reflection on Poncelet's introduction of ideal relations in geometry, and the reconceptualization that Chasles offered for these "ideal elements" in his 1837 *Aperçu historique* played a key part in Kummer's introduction of "ideal complex numbers." This part is clearly perceptible in the structure of the 1846 publication, and I will explain how we can read the effect of Chasles' reconceptualization in the definitions that Kummer presents. I also argue the parallel between ideality in projective geometry and in Kummer's work on numbers helps us understand features of the "ideal complex numbers" that have puzzled historians. This episode is interesting at a higher level, since it suggests that the philosophical reflection on the value of generality that geometers like Poncelet and Chasles developed in the context of the shaping of projective geometry was instrumental *as such* in inspiring key conceptual developments in other domains of mathematics, and precisely in this case, the introduction of ideal elements more widely in mathematics.

References:

Boniface, J. (2016) 'A process of generalization: Kummer's creation of ideal numbers', in *Handbook on Generality in Mathematics and the Sciences*, eds. K. Chemla, R. Chorlay and D. Rabouin. Oxford, Oxford University Press: 483-500.

Chemla, K. (2016) 'The Value of generality in Michel Chasles's historiography of geometry', in *Handbook on Generality in Mathematics and the Sciences*, eds. K. Chemla, R. Chorlay and D. Rabouin. Oxford, Oxford University Press: 47-89.

Edwards, H. M. (1977) *Fermat's Last Theorem. A genetic introduction to Algebraic Number Theory*. Graduate Texts in Mathematics. New York, Berlin, Heidelberg: Springer-Verlag.

Parallel Session 3, 11:45h – SESSION A

P. Seban - *From Zeno ad infinitum, iterative reasoning in early Greek mathematics*

We are interested in some aspects of the very early mathematical conception and use of the infinite in Ancient Greece. Following a suggestion of Fabio Acerbi (2000), we examine the extant corpus of and about Zeno of Elea in the context of the practice of reasoning *ad infinitum* (*eis apeiron).* Zeno introduces in philosophy what could be called "iterative reasoning", which is based on the a priori recognition that a certain operation, when performed, will ineluctably replicate the conditions for it to be performed again (in the way a continuous magnitude is always cut in new cutable continuous magnitudes, or a number is always followed by a followable number). Just as classical *reductio ad absurdum*, iterative reasoning secures its conclusion thanks to a meta-logical step back that in the present case acknowledges the unachievability of the operation. Complete and incomplete inductions, as well as *reductio ad infinitum* or infinite descent are all conceptually different arguments, to be found both in early maths and early philosophy, all based on such iterative reasoning. The Aristotelian analysis of the infinite in *Physics*, III, reflects the vivid difference between this conception of infinity as logical unachievability and the more substantial conceptions of infinite objects or multiplicity to be found in the Presocratic and Atomistic traditions. Following again the lead of Fabio Acerbi, we want to reflect on the way these conceptual differences allow or forbid certain of the aforementioned concepts or logical moves to be introduced and maintained in early mathematical practices.

B. Wilck - *Euclid's definitions*

In the *Topics*, Aristotle puts the following three putative definitions of *even* and *odd* to the test, to the extent that no other definition of these two arithmetical terms seems even possible:

> *odd* $=_{\text{def}}$ *the state of a number which is greater by a unit than an even number;*[1] *even* $=_{\text{def}}$ *the state of a number which is divisible into halves;*[2] and *odd* $=_{\text{def}}$ *the state of a number having a middle*.[3]

In Euclid's *Elements*, variants of the same definitions of *even* and *odd*, which Aristotle expressly rejects about a century before Euclid, are stated:

> *even number* $=_{\text{def}}$ *the number which is divisible into halves;*[4] *odd number* $=_{\text{def}}$ *the number which is not divisible into halves;*[5] and *odd*

[1] *Top*. VI.4, 142b7–10.
[2] *Top*. VI.4, 142b12–19.
[3] *Top*. VI.12, 149a29–37.
[4] Eucl.*El*.VII.def.6.
[5] Eucl.*El*.VII.def.7i.

number $=_{\text{def}}$ *the number which differs from an even number by a unit.*[6]

Even though Euclid's definitions fail Aristotle's dialectical tests, they are nonetheless deductively employed as explanatory premises in mathematical proofs in *Elements* IX–X.[7] Thus, it seems that Aristotle's dialectical tests in the *Topics* are not successful or that there is a problem with their applicability to mathematical practice. My paper argues for the following solution to this problem: Aristotle's dialectical refutations are directed towards the concessions of a Platonic interlocutor whose meta-mathematical commitments cannot be ascribed to Euclid.

Parallel Session 3, 11:45h – SESSION B

J. Pérez Escobar - *Mathematical modelling in biology and non-materialistic idealizations: the resilience of the gap between biology and physics*

It is often considered that the introduction of mathematical methods in biology, such as mathematical modelling, contribute to its maturity as a science and brings it closer to materialistic sciences like physics. However, I claim that non-materialistic teleological notions characteristic of explanations in biology often make their way into mathematical models of biological phenomena. Their introduction is facilitated by the particularities of the practice of modelling in biology, which deviates from its physical counterpart. Understanding how scientists of different backgrounds use mathematics to represent empirical phenomena and how they interact at the intersections of their disciplines is a prerequisite for a proper epistemological understanding of mathematical modelling.

R. Kelter – *Markov-Chain-Monte-Carlo for Hypothesis Testing – An alternative to p-values with regards to the replication crisis of medical studies*

Data and quantitative information as well as the conclusions drawn from them have more influence in todays science than ever before. In a more and more digitalized world the question arises, how these conclusions are calculated and derived. Therefore, inferential statistics as a mathematical discipline, especially statistical inference models as a mathematical concept are investigated, analyzed and compared in this paper. An emphasis is put on the comparison of the classical frequentist statistics - influenced in big parts by the work of R.A. Fisher - and the contrary bayesian approach. While the later one is older, the bayesian inference is experiencing a renaissance in the last years due to several advantages over the classical approach and the availability of powerful computing ressources and algorithms. While the classical frequentist model has multiple conceptual problems, it is still todays widely used inference model. The bayesian approach has conceptual advantages but on the other hand lacks in terms of efficiency and has several algorithmic pitfalls. By discussing the question how the mathematical

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[6] Eucl.*El*.VII.def.7ii.

[7] See Eucl.*El*.IX.21–34 (Euclid's theory of even and odd numbers); cf. Eucl.*El*.X.28.lems.1–2; X.117 (Euclid's theory of incommensurable lines).

concept of statistical inference has transformed over time, and by sketching the evolution of both inference models, the article gives valuable insights into the history of one of the most important mathematical concepts. The results show, that there is much to investigate especially regarding the history and evolution of the bayesian approach and its predominant class of Markov-Chain-Monte-Carlo algorithms.

Parallel Session 4, 14:30h – SESSION A

M. J. García-Pérez - *The prehistory of geometry: Early China as a case study*

Some authors have declared that the early history of mathematics could be divided in three main periods: i) proto-geometry or prehistory of geometry; ii) basic geometry oriented to practical goals; and iii) geometry in a proper sense. While some works on the history of mathematics recognise this tripartite division, little attention to the prehistoric data is payed.

The emergence of proto-geometric knowledge coincides with the observance of some necessary conditions. These are related with the transition from the Palaeolithic to the Neolithic period. Moreover, the most important features of this new era are i) the transition from nomadic to sedentary communities, allowing populations to grow and to have some leisure; ii) the division, organization and specialization of labour; and iii) the use of tools that enhance our cognitive abilities.

Particularly, it is important to notice that some new kingdoms needed to perform some activities in order to stablish a common ideology to agglutinate large and unrelated groups of people. One of these activities was the development of proto-astronomical knowledge, the acknowledgment and correct interpretation of astronomical phenomena. Here, some “mathematical” tools and some basic geometric concepts –square and circle, mainly– were developed.

China, among other civilizations, offers relevant material to analyse this issue. For example, it can be seen that in order to accurately perform some rituals in the Liangzhu Culture, in the Zhou dynasty (3300-2200 b.c.e.), the development of a reliable calendar was needed. To this purpose, a *biao*, or gnomon, was used. Furthermore, some cities in the Longshan culture (3000-2000 b.c.e.) were planned according to some astronomical phenomena, building the Taosi Observatory to find the correct place to situate the capital city.

Therefore, in order to perform correctly these rituals, or to build accurately the cities –essential ingredients to maintain a strong sense of community and celestial blessing by their chiefs– a more sophisticated, theoretically oriented use of spatial relations was required. Consequently, here can be situated the genesis of proto-geometry.

T. Hirth - *From Pebbles to nimbers and stars*

Around 1506 Luca Pacioli wrote his *De Viribus Quantitatis*, a compendium of mathematical recreations, a cornerstone in the history of recreational mathematics. One example from this work is the description of a two player game in which players alternate adding a bounded number of beans to a pot until this is full. The player that actually fills the pot, wins.

This is the oldest version we know of a game in the family of games first studied mathematically by Bouton in 1901. Later work by Sprague and Grundy, in the 1930s established the foundation for "Combinatorial Game Theory", an active area of research akin to combinatorics. John Conway in 1972 and Conway, Berlekamp, and Guy in 1982 published the main texts, still very influential.

This is just one of many examples of a recreation giving rise to a "serious" field of mathematics. Many others are better known, bringing names as Euler and Hamilton to mind. However, we will focus on this pioneer work of Fra Luca and try to show its relevance in the arising of several mathematical concepts.

M. Chopra - *Before or beyond concepts*

I propose to present a work in progress. I study the intellectual backgrounds of Assyro-Babylonian mathematical astronomy.

I think, this seemingly pre-conceptual science and its formulation is highly interesting because, among other reasons, we can observe the thoughtful development of very elementary procedures and objects in contrast with traditions where these objects are already given.

I would like to present the questions at stake in three points:

1. There are *schemes*, that are together what we would call principals and concepts, that are never exposed as such, though being active everywhere. Their expression oscillates between important terms that, so to speak, don't appear in more "technical" texts and structure of procedure's that are themselves not explained in these texts. I will try to present two of them: symmetry and itermeasurability.

2. A triangle that one can draw between three grossly defined fields – general concerns, astronomy and mathematics – helps us to understand these scheme’s meaning: we can hardly perceive them if we take one of these fields separately but they are clearly perceptible if we consider the relation between these fields. For instance, symmetry could appear as an ad hoc trick in mathematics but doesn't any longer when studied through the triangular relation in which it is reflected.

3. These schemes have, I would argue, also not to be understood by themselves or abstractly but they permit to shape the observation, taken in a broad sense. Thus, they are a background on which some phenomena can be regarded as signs. Thus, the phenomena that occurs around the full moon keep, according to my hypothesis, a mathematical and a non mathematical meaning.

In one way, we can treat those *schemes* historically and see them as the ancestors of our *concepts* but in another way, they are wider and more open, and, as such, still require to be meditated from us today.

Parallel Session 4, 14:30h – SESSION B

J. Zeman - *Hilbert's Grundlagen der Geometrie in context of non-Euclidean geometries*

In this address, we will present topics from the second and third chapter of Hilbert's *Grundlagen der Geometrie* from 1899. We will discuss Hilbert's proofs of the independence of the particular axiom groups, mainly of the Parallel Axiom.

It was as early as in 1894 when Hilbert in his lectures on the foundations of geometry used the axioms from Pasch's *Vorlesungen über neuere Geometrie* (1882) and divided them into groups according to the relations between the elements of the geometry. The single Parallel Axiom was introduced only at the very end so that the Euclidean geometry built a closure of the non-Euclidean geometries and, fully in accordance with the common Klein's treatment, all these geometries were considered as special cases of the projective geometry. Another result of these Hilbert's lectures was his study of the theorem of the straight line as the shortest connection of two points.

After a while, Hilbert returned to the foundations of geometry (Euclidean now) in his lectures from 1898/1899 and in the following first edition of his book *Grundlagen der Geometrie*. In the lectures, he aimed on analysing its features, in the book, he stressed more its construction and because he didn't introduce the ideal elements there, the connection with the projective geometry got lost altogether. The Parallel axiom served him mainly as a tool for the proof of the elementary Pappus (called Pascal by Hilbert) and Desargues theorems. As a part of these lectures, he also introduced the common history of the Parallel axiom based partially on the presentation from Veronese's *Grundzüge der Geometrie* (German edition 1894).

S. Shokrani - *Leonard Nelson's Philosophy of mathematics and its application*

The German philosopher and pedagogue, LEONARD NELSON (1882-1927), had the reputation to be the philosopher of the Hilbert's Programme. He contributed in establishing foundations of mathematics and logic, as a member of the circle around DAVID HILBERT (1862-1943). For example he published a joint work with the mathematician, KURT GRELLING (1886-1942), which included a generalized form of Russell's Paradox.

Following his philosophy of mathematics, he conceptualized and developed a method of teaching. In my talk, I will shortly introduce his philosophy of mathematics, which was on the philosophical line of IMMANUEL KANT (1724-1804) and JAKOB FRIEDRICH FRIES (1773-1843). Based on that, his pedagogical conception will be discussed.

T. Schütz - *Exploring gravitational lensing*

My contribution will discuss the idea of gravitational lensing from a historical point of view. The basic idea of gravitational lensing has a long and interesting prehistory which goes back to early manuscript notes by Einstein dating from the year 1912. Historical research has established that the idea of gravitational lensing and its immediate consequences were forgotten and rediscovered time and again. Recently, we have analyzed four sheets of related calculations which are part of a batch of largely unidentified notes and calculations by Einstein. He derived a quadratic equation for the angle under which an observer can see the light emitted by a distant light source behind a massive object that acts as a lens. The derivation uses geometrical optics and some idealizations such as point-like objects and the assumption that the light ray is bent at one single point. The two solutions of the quadratic lensing equation correspond to a double image that is seen by an observer. This basic lensing equation together with the concepts of geometrical optics opens a space of implications that can be explored along different dimensions. We argued that Einstein explored the idea along different pathways in this space of implication, and that these explorations are documented by different calculational manuscripts.

Parallel Session 5, 16:45h – SESSION A

S. Pietroni - *Equation of time in Alfonsine astronomy*

Throughout time not very many astronomical tables kept evolving. However, some did, as is the case of the table for the equation of time, where the parameters underlying it were often modified but the format of the table remain basically unchanged for many centuries. In this presentation, we offer a general survey of the evolution of the equation of time through different traditions, starting from Greece with Ptolemy arriving to the pre-Copernican period, analyzing from manuscripts ancient and medieval texts and tables. Alfonsine astronomy provides examples of the equation of time, which will be the object of this presentation.

Alfonsine astronomy that was born in Toledo under the patronage of King Alfonso X and that flourished in Europe from the second half of the 13th century to the middle of the 16th century.

The aim of my work is to find out how this kind of table was computed and confirm its authorship, when possible, in order to better understand the historical milieu in which the table has been improved and its diffusion.

This research will contribute also to a better understanding of the technical knowledge of mathematical methods for astronomy and their evolution in time from Antiquity to the 16th century.

G. Loizelet - *Sun-Earth distance: Al-Biruni facing a correct calculation method that cannot be trusted*

I will first present a method used by greeks and arabs for the determination of Sun-Earth distance based on Moon and Sun observable phenomena.

Next I will shortly present N.M. Swerdlow 1969 analysis showing that although this method was theoritically correct, its excessive sensibility to parameters and roundings makes it inoperable.

Lastly, I will focus on how Al-Biruni (Khwarezmian 11th century polymath), while aware of this issue, deals with it in chapter X.6 of his 1030 treatise *Al-Qanun al-Mas'udi.*

Parallel Session 5, 16:45h – SESSION B

M. Friedman - *Braids, diagrams, models and the concepts of 'Anschaulichkeit' and 'Intuition'*

Starting 1925 a flourishing of the mathematical investigation of braids took place – especially within group theory. This was prompted mainly due to Emil Artin's 1925 paper "Theorie der Zöpfe", which was later revised and published in 1946 as "Theory of braids". The treatment of Artin attempts for the first time to achieve an algebraic formulation of the braid group: his aim is explicitly "to arithmatize", i.e. to present symbolically, with the tools of group theory, braids. However, when considering more closely Artin's paper, some of the algebraic arguments presented by him can be also seen by looking at the diagrams drawn all along the paper. When considering the fact that Artin talks often about the "geometrical meaning" of his research and that his arguments are described as "anschaulich" [visual, illustrative, intuitive], one has to ask what was the meaning of this term: "Anschaulichkeit", when considering the fact that Artin meant to reduce his arguments to purely symbolic-syntactical ones.

Moreover, during the 1930s-1950s, Oscar Chisini and his students also investigated braids, though now within the context of algebraic geometry and complex curves. But in contrast to Artin, Chisini emphasizes that one should use material models, made from, for example, thick threads. Chisini emphasized that these instruments were meant to concretely visualize braids, and called these models were made to be clear for the "visual intuition". Taking braids as a case study, the questions I would like to approach in my talk concern the relations between these two types of "visual intuition" resp. "Anschaulichkeit": how these concepts were dependent on the two-dimensional diagrams or on the three-dimensional models? Or on the mathematical cultures present in Italy and in Germany? And how did the concepts develop when the research on the braid group became more and more algebraic?

D. Waszek - *On the use of the concepts of ‘representation’ in the history of mathematics*

In keeping with this year's theme, I would like to examine the use of the concept of “representation” in the history of mathematics. One often sees the word used to describe how sentences, diagrams, various kinds of formulas, and so on appear to be used by mathematicians as various ways to access the same objects. Yet this raises difficulties, which have led some researchers – Ken Manders being a prominent example – to argue that we should stop using it altogether. Why is the concept problematic, and should we ban it? I would like to approach this question through a case study borrowed from the history of the “calculus of operations” – that is, the algrebraic manipulation of differentials and integrals as operators – in the late XVIIth and in the XVIIIth century. First, it immediately appears in this history that sometimes, our authors do not know what their symbols are referring to, or hold false or inconsistent views about this – which should make us wary about describing them as “representing” something. Second, and more importantly, we see that the introduction and exploration of new “representations” – in our case, new notational devices – leads to substantial changes in the subject matter of the enquiry: new objects are progressively introduced and previous symbolic manipulations are re-interpreted in new terms. This, again, pleads against talking about “representations” of something fixed. Yet I will argue that eliminating the concept of representation risks making our authors' mathematical practice incomprehensible: we need it to describe what our authors' think they are doing, even while keeping in mind that, from a retrospective point of view, we might find that their understanding of what and how they are representing is not satisfactory.

Parallel Session 6, 9:30h – SESSION A

B. Stenhouse - *Embracing Nature in Formulae: The Hidden Mathematics of Mary Somerville*

In the 1830's, after the publication of her first book *Mechanism of the Heavens*, Mary Somerville was known throughout the UK and continental Europe as an expert in analysis, and its applications to astronomy. Her next work centred on the theme of 'the physical sciences', which she claimed were united by the "bond of analysis" which would "ultimately embrace almost every subject in nature in its formulae". However, these formulae are conspicuously missing from both this and all her future publications.

An unusual situation is made ever more peculiar by the existence of two unpublished manuscripts Somerville completed in 1834, both of which are explicitly mathematical and would have slotted in perfectly to her analytical agenda. In my talk I will outline and contextualise the content of one of these papers, and open the question of why mathematics vanished from Somerville's published works.

A. Kiel Steensen - *Dedekind's articulation of the Modulgesetz. A case study of a semiotic process in mathematical practice*

In this talk I will present a case study of representations developed by Richard Dedekind (1831-1916) from an early occurrence of modules in [1871] to the Modulgesetz in [1897]. The study is a part of my PhD project that tries to describe: (1) how mathematical practices produce and interact with concrete representations, and (2) whether constructing new representations or using existing representations in new ways produce and change mathematical reference.

Dedekind's approach to mathematics is often described as conceptual[1]. However, Dedekind's Nachlass is full of manipulations with concrete signs, such as working out examples, stepwise generalizations, systematic variations, etc. The case study examines what role these calculations play in the articulation of the Modulgesetz.

The case study then asks whether the process of developing the Modulgesetz can be seen as the articulation of a signifying unit, which, as the theory develops, initiates the distinction between objects that do and do not satisfy the Modulgesetz. If the answer is yes, then concrete interactions with mathematical representations has affected the ontology of contemporary lattice theory.

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1 For instance: '[Dedekind had] a clear inclination to address mathematical problems by radically reformulating the whole, relevant conceptual setting. Dedekind ascribed to the systematic introduction of new concepts a central role in the solution of problems and in the clarification of existing mathematical knowledge.' [Corry 2004: 66].

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Parallel Session 6, 9:30h – SESSION B

D. Kant - *Forcing as a Part of Set-Theoretic Practice*

Forcing is a part of set-theoretic practice since more than fifty years. Much set-theoretic knowledge of today is based on forcing. The main problem in the philosophy of set theory –the lack of explaining the status of the numerous, provably undecidable sentences– is as severe as the forcing method is successful in application.

In order to approach the philosophical independence problem, the role of forcing in set-theoretic practice is examined. I aim at a description of the current role of forcing by interviewing expert set theorists and taking into account the historical developments.

In my talk, I argue briefly for my choice of a practical approach, and in a second part, I examine different aspects of the role of forcing: The quantitative use of forcing and the acceptability or naturalness of forcing axioms.

The rest question how often forcing is used in practice is fundamental to determine the importance of the forcing method for the set-theoretic discipline. Some set theorists or philosophers have the view that every set theorist works with forcing.[1] However, there is a branch of set theory (apart of descriptive set theory) which seems to be rather untouched by the independence phenomenon. I describe what set theory is for them.

The second question how acceptable or natural forcing axioms are for set theorists is important to describe how the independence problem looks like for the set theorists themselves. In the realm of independence, it seems (at the moment) impossible to get a grip on truth, neither on provability, but we can get a grip on the acceptability and naturalness of new axioms seen by the set theorists who work with them. I describe the views of some set theorists on these notions, using a comparison to large cardinal axioms.

[1] This is for example a prominent argument for the multiverse view used by Hamkins. By the vast presence of the forcing method in practice, he argues that the universe view is completely missing out the actual situation.

M. Imocrante & M. Osta - *Conceptual change and (Philosophy) of mathematics*

The attention of the philosophical community has been repeatedly drawn on the existence of phenomena of conceptual change in mathematics (Lakatos 1976, Gillies 1992, Tanswell 2017). However, the dominant view of definitions of mathematical concepts and theories (Russell, Whitehead 1910) tends to make it difficult to deal with cases in which a given mathematical term, which is taken to refer to a given object, is found to have different meanings in different contexts of use. Moreover, most theories of conceptual change in science are not suitable for mathematics: this is mostly because they are motivated by some philosophical concerns that do not arise in this discipline (e.g. the commitment to scientific realism, or the intention to preserve the stability of the reference for natural kinds terms).

The present work aims at clarifying a suitable semantic framework to understand conceptual change within mathematics. In particular, we are going to investigate which are the conditions that a theory of meaning (and reference) has to meet in order to account for cases in which a mathematical term seems to change meaning through the historical development of the discipline. Our cases studies will be drawn from the history infinitesimal calculus (Boyer 1949, Salanskis 1999). We thus argue that reference should not be considered as a central semantic feature for mathematical terms. Instead, we suggest to understand the meaning of those terms as emerging from the collective cognitive practice of the mathematicians' community through times, their central semantic properties being inferential and operational.

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Tanswell 2017, "Conceptual Engineering for Mathematical Concepts", Inquiry.

C. J. Rittberg - *Intellecual Humility in Mathematics*

What is intellectual humility in mathematics? The mathematician who submits herself to the forces of a formal framework cultivates a meekness that aligns with old Christian interpretations of humility. Yet formal frameworks have been fruitfully changed and expanded by mathematicians.

Because mathematical practices have histories, "mathematics" is a moving target. "Humility" gives in similar fashion to the forces of history: it has been vice and virtue, interpreted in conflicting ways and its perceived importance has risen and fallen. The aim of this talk is to highlight aspects of mathematical practices and intellectual humility in a way that fosters an understanding of what intellectual humility in mathematics amounts to.

Parallel Session 7, 11:45h – SESSION A

J. L. Gastaldi - *The emergence of a mathematical concept of "symbol" in the work of Robert Woodhouse*

The concept of "symbol" plays an important role for historians and philosophers of mathematics since it is to the "symbolic revolution" in mathematical thought that are usually attributed many of the fundamental developments that led to modern mathematics. However, this widespread use of the concept of symbol is in most cases both anachronistic and extra-mathematical. Indeed, the notion of symbol, and the sense of homogeneity in mathematical writing that it conveys, was alien to mathematicians like Vieta and Descartes, and it remained unfamiliar to mathematicians throughout the entire 17th and 18th centuries, who significantly made use of a multiplicity of other non-interchangeable terms (such as "letter", "species", "character", "sign" or "expression"). Against this uncritical use, I will inquire into the history of the concept of symbol, as a mathematical concept, that is, as a concept intended to have a specific mathematical content, explicitly introduced by working mathematicians themselves as a way to cope with singular mathematical problems. I will claim that its introduction only took place in the framework of the elaboration of an abstract (or "symbolical") algebra by the English algebraic school during the first half of the 19th century. In particular, I will present the work of Robert Woodhouse (the "father" of the English algebraists) to show how a concept of symbol became necessary to give new life to the algebraic foundations of analysis in the wake of the failure of the Lagrangian program. After showing the difficulties of such a task and the strategies that determined the program of the English algebraists, I will point out both the strengths and the inherent limits of that specific symbolical approach for the foundational role it was expected to have.

N. Michel - *Numbers as concepts & Numbers as symbols*

While Hermann Schubert (1848-1911) is nowadays mostly known for his pioneering work in enumerative geometry, he also devoted a large part of his career to the writing of mathematical recreations, handbooks and philosophical reflections, mostly on algebra and arithmetic.

Consequently, in 1898, he received the honor of being asked to take part in Felix Klein's and Wilhelm Meyer's Encklopädie der mathematischen Wissenschaften, of which he wrote the very first chapter, on the foundations of arithmetic. But a few months later, this article was subject to a ferocious and scornful review by none other than Gottlob Frege (1848-1925). Whereas Frege, in his own foundational writings, attempted to obtain and clarify the concept of number by determining what numerical identity consists in, Schubert rooted arithmetical practice within the very act of counting, and within a formal use of symbols justified by Hermann Hankel's principle of permanence.

This controversy, we will claim, stems from two incompatible accounts of what properly constitutes a mathematical object. We will show how this divide, far from being restricted to the concept of number, actually encompasses, and perhaps even follows from, important discussions pertaining to the constitution of geometrical objects. Building on an observation made by Mark Wilson, we will show how the discussion between Schubert and Frege can serve as a vantage point from which a better understanding of how mathematical concepts grow and thrive can be gained.

Parallel Session 7, 11:45h – SESSION B

S. Decaens - *Lattice theory, abstract algebra and abstraction in the 1930s*

From the 1930s, the concept of lattice became the object of a mathematical theory, particularly promoted by US researchers. In my talk I would like to link together the mathematical practices related to lattices and different aspects of the *theory*. First, we will see how the concept of lattice was inserted into a specific mathematical context. In different discourses, abstract algebra appears as a new trend, in which lattices are both an object of study and a tool. Secondly, we will focus on epistemic values, such as abstraction, supporting the use of lattices. Here, abstraction has a specific meaning related to the undefined nature of the objects constituting a lattice, which permits to adopt a general point of view. The combination of abstract algebra and abstraction will allow us to discuss the way actors understand, use and structure lattice theory.

M. Chalmers - *Georges Bouligand's concept of direct methods in mathematics*

A direct method in mathematics, for Georges Bouligand (1889-1979), is, firstly, one that reveals the reason why behind a result. A direct method deals directly with the problem or object studied in a way that preserves contact with intuition. Bouligand views the creation of direct methods as a major trend in the mathematics of his time:

> Tout semble indiquer, dans l'évolution des mathématiques, un acheminement progressif vers les méthodes directes. La causalité apparaissant de plus en plus clairement, les paradoxes tendent à s'éliminer. [Bouligand, 1933]

Bouligand attributes this trend to the rise of the axiomatic approach in mathematics and thus sees direct methods as marrying logic with intuition. He sees the direct method in the calculus of variations as one example of his notion of direct method and argues that the concept can be extended to diverse areas of mathematics, while remaining united by certain common characteristics, including notably the use of mathematical groups as well as certain concepts in topology. The concept of direct method is strongly present in Bouligand's own mathematics, first becoming apparent in his work on the Dirichlet problem, from the early 1920s and most centrally in his later theory of géométrie infinitésimale directe.

This presentation will explore Georges Bouligand's notion of direct method as it appears in his own mathematical and philsophical works and will attempt to go some way in tracing the possible origin and development of this concept.

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Plenary Lecture, 14:30h

J. Ferreirós - *Investigating long-term developments in the history of mathematics: The case of the function concept*

In recent years there has been interest in the topic of long-term history, after a time of concentration in very detailed and local case studies. I shall discuss problems and methods having to do with one concrete type of historical question that demands a long-term perspective: the emergence and development of the function concept over a period of several centuries before 1900. The question is prominent and has been studied many times, but there are aspects open to substantial disagreement, and also some important aspects insufficiently researched. Hence the need to talk about problems and methods. In order to avoid remaining at a too general level, I will also discuss one particular case study: G. P. Lejeune Dirichlet's notion of 'arbitrary function', and the methodological reasons why he was led to start a tradition of 'conceptual mathematics' (begriffliche Mathematik).

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